

Characterization of the transition from defect- to phase-turbulence

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Abstract

For the complex Ginzburg-Landau equation on a large periodic interval, we show that the transition from defect- to phase-turbulence is more accurately described as a smooth crossover rather than as a sharp continuous transition. We obtain this conclusion by using a powerful parallel computer to calculate various order parameters, especially the density of space-time defects, the Lyapunov dimension density, and the correlation lengths of the field phase and amplitude. Remarkably, the correlation length of the field amplitude is, within a constant factor, equal to the length scale defined by the dimension density. This suggests that a correlation measurement may suffice to estimate the fractal dimension of some large homogeneous chaotic systems.

47.27.Cn, 47.27.-i, 47.27.Cn, 05.45.+b, 05.70.Ln, 82.40.Bj

Recent advances in laboratory technique [1] and in computer simulation [2–5] have opened up the study of boundary-independent spatiotemporal chaos in large homogeneous sustained nonequilibrium systems [6]. Many fundamental questions remain unanswered about such chaotic systems, e.g., what different states can occur, how transport depends on different states, and what kinds of bifurcations separate one state from another. An especially interesting question is whether ideas from statistical mechanics might be applicable to chaotic nonequilibrium systems in the thermodynamic limit of infinite system size [7–9].

A significant step towards understanding some of these questions was recently reported by Shraiman et al [2]. These researchers studied different spatiotemporal chaotic states of the one-dimensional complex Ginzburg-Landau equation

$$\partial_t u(x, t) = u + (1 + ic_1)\partial_x^2 u - (1 - ic_3)|u|^2 u, \quad x \in [0, L], \quad (1)$$

on a large periodic interval of length $L = 1024$, which they assumed to be large enough to approximate the thermodynamic limit of infinite system size. Here the variables t and x denote time and position respectively, the complex-valued field $u(x, t) = \rho e^{i\phi}$ has magnitude $\rho(x, t)$ and phase $\phi(x, t)$, and the parameters $c_1 > 0$ and $c_3 > 0$ are real-valued. Eq. (1) is an important model of spatiotemporal chaos because it is simple, experimentally relevant [10] and universal [9]: spatially extended systems that undergo a supercritical Hopf bifurcation from a static to oscillatory homogeneous state reduce to Eq. (1) sufficiently close to the onset of the bifurcation. Interesting dynamics are predicted and are observed beyond the Newell line $c_1 c_3 = 1$ since all plane wave solutions of Eq. (1) are linearly unstable to the Benjamin-Feir instability for $c_1 c_3 > 1$ [9].

Shraiman et al summarized their simulations in the form of a phase diagram in the c_1 - c_3 parameter plane (Fig. 3 of Ref. [2]). Based mainly on calculations of the density of space-time defects n_D [11], this diagram showed continuous and discontinuous transition lines (analogous to second- and first-order phase transitions) separating defect-turbulent from phase-turbulent states [11]. Of special interest to us is the continuous chaos-to-chaos transition line L_1 in their Fig. 3, which occurs for $c_1 \geq 1.8$. It is somewhat mysterious why

the density n_D decreases to zero at an L_1 line that is distinct from the Newell line: in the limit of infinite system size and of infinite time, what prevents defects from forming anywhere to the right of the Newell line ($c_3 > 1/c_1$)? The mystery of the L_1 line can be partly appreciated by trying to reason by analogy to equilibrium statistical physics. Assuming that the chaotic fluctuations of Eq. (1) act as a finite-temperature ergodic noise bath and observing that the derivatives in Eq. (1) represent short-ranged interactions between different parts of the field u , we would not expect distinct phases at finite temperature in one-space dimension [12].

Because so little is known about possible critical phenomena of large homogeneous nonequilibrium systems and because Eq. (1) is such an important model, we have tried to characterize more carefully the dynamics near the L_1 line for the fixed parameter value $c_1 = 3.5$. By calculating various order parameters over length scales as large as $L \leq 10^6$ and over time scales as large as $T \leq 10^7$, we are able to show below that the change from defect- to phase-turbulence near the L_1 line is more accurately described as a smooth crossover rather than as a sharp continuous transition with power-law scaling of order parameters [2]. It is then a possibility that phase turbulence (i.e., a chaotic state with $n_D = 0$ in the thermodynamic limit) does not exist although we can not settle this with our present computer resources. Our calculations also confirm several points predicted by Shraiman et al and by other researchers [13], e.g., that the spatial correlation function of the phase should decay exponentially inside the phase-turbulent regime. We have also studied whether the dimension density δ (Lyapunov fractal dimension per unit volume) is a useful order parameter for characterizing changes in spatiotemporal chaotic states [9,3,14]. The dimension density defines a dimension correlation length $\xi_\delta = \delta^{-1/d}$ [9] which is the characteristic size of dynamically independent subsystems of spatial dimensionality d [3]. A comparison of ξ_δ with other characteristic length scales as a function of the parameter c_3 gives the remarkable result that ξ_δ is, up to a constant factor, equal to the spatial correlation length of the field magnitude, ξ_ρ , from the Newell line to beyond the L_1 line.

An important resource for the advances reported below was a CM-5 parallel computer

[15], which facilitated the study of much larger space-time and parameter regions than were previously conveniently accessible. Our numerical methods for integrating Eq. (1) and for calculating related order parameters are similar to previous methods [2] except for modifications of algorithms and codes to take advantage of the CM-5's scalable parallel architecture [3]. For most of our simulations, we used a time step $\Delta t = 0.05$ and a spatial resolution of two Fourier modes per unit length; this space-time resolution was dictated by the need to detect isolated space-time defects when estimating the density n_D [2]. Our initial condition for most runs was $u(x, t = 0) = 0.4 + \eta$ where η was a uniformly distributed δ -function correlated random variable in the interval $[-0.02, 0.02]$. We typically averaged over 64 or more random initial conditions spanning an integration time of 2×10^5 so that the total effective integration time was perhaps as long as $64 \times (2 \times 10^5) = 1.3 \times 10^7$ time units.

Before discussing our results, we note that for $c_1 = 3.5$, for an integration time of $T = 10^5$, and for a periodic interval of length $L = 1024$, Shraiman et al argued the existence of the L_1 line using two key observations [2]: (1), that the density n_D vanished as a power law $n_D \propto (c_3 - \bar{c}_3)^\alpha$ with exponent $\alpha \approx 2$ and with $\bar{c}_3 \approx 0.77 > c_3^{\text{Newell}} = 1/c_1 = 0.286$; and (2), that the correlation time τ of phase fluctuations [16] diverged as a power law also at \bar{c}_3 , as the inverse of the defect density, $\tau \propto 1/n_D$. For $c_3 < \bar{c}_3$, Shraiman et al observed a less-disordered phase-turbulent regime with n_D empirically equal to zero and with slower-than-exponential decay of temporal correlations [2]. If defects do not occur in the thermodynamic limit, a perturbation theory in the small quantity $\epsilon = c_1 c_3 - 1$ yields a simpler description of phase turbulence near the Newell line, $\epsilon \rightarrow 0$. In that limit, Eq. (1) reduces to the Kuramoto-Sivashinsky (KS) equation [17,9]

$$\partial_t \phi = -\epsilon \partial_x^2 \phi - \frac{1}{2} c_1^2 (1 + c_3^2) \partial_x^4 \phi - (c_1 + c_3) (\partial_x \phi)^2, \quad \epsilon = c_1 c_3 - 1, \quad (2)$$

and the amplitude ρ becomes an algebraic function of a spatial derivative of the phase,

$$\rho \approx 1 - \frac{c_1}{2} \partial_x^2 \phi. \quad (3)$$

Some of our calculations below provide the first quantitative comparisons of phase turbulence as described by Eqs. (2) and (3) with phase turbulence as empirically observed in Eq. (1).

For the parameter value $c_1 = 3.5$, for a system size $L = 4096$, and for an effective integration time of $T = 10^7$ (after allowing transients of duration 10^4 to decay), we find in Fig. 1 that n_D is finite substantially to the left of the L_1 line as calculated in Ref. [2]. Far to the right side of the L_1 line, our data in Fig. 1(a) approximately reproduce the previously reported [2] power-law scaling with exponent $\alpha \approx 2$. Closer to the L_1 line, a least-squares fit of the three-parameter expression $a(c_3 - \bar{c}_3')^\alpha$ to the nine left-most points gives a much larger exponent $\alpha \approx 6.8$, with an onset of phase turbulence ($n_D = 0$) at $\bar{c}_3' = 0.74 < \bar{c}_3 = 0.77$. Assuming equal errors bars on all data points, we find the chi-square value for the fit to be $\chi^2 = 4.6 \times 10^{-12}$. The increase in the exponent with increased space-time resolution suggests that a power-law scaling is inappropriate. As shown in Fig. 1(b), we find a better fit of the same data with the functional form

$$n_D = a \exp \left(-b / (c_3 - \bar{c}_3'')^\alpha \right), \quad (4)$$

which is the expected behavior for thermodynamic Gaussian fluctuations of the phase gradient $\partial_x \phi$ if large values of the latter are the reason for defect nucleation [2]. If we set $\alpha = 1$, a least-squares fit of Eq. (4) to the nine left-most data points yields the three parameter values $a = 0.66$, $b = 0.98$, and $\bar{c}_3'' = 0.70 < \bar{c}_3' = 0.74$ with a $\chi^2 = 8.1 \times 10^{-13}$. If we set $\bar{c}_3'' = c_3^{\text{Newell}} = 0.286$ to test whether Eq. (4) is consistent with the onset of phase turbulence at the Newell line, a least-squares fit (again to the nine left-most points) gives the parameter values $a = 0.018$, $b = 0.017$, and $\alpha = 8.8$ with a substantially poorer $\chi^2 = 8.2 \times 10^{-11}$. Our data spanning the crossover region evidently lie too far to the right of the Newell line to determine whether the defect density goes to zero before or at this line.

To test independently the important implication of Fig. 1 that a crossover occurs (so that phase turbulence may not be a distinct phase from defect turbulence), we have calculated other order parameters over the same parameter range. The occurrence of a crossover is supported by Fig. 2, which summarizes correlation times τ of the phase $\phi(x, t)$ [16] as a

function of c_3 . Fig. 2(a) shows that τ does not diverge to infinity at the L_1 line (as reported in Ref. [2]) but instead is large and finite a little bit to the left of the L_1 line. We find that time correlation functions of the phase decay exponentially down to at least $c_3 = 0.77$ but do not have a simple functional form for smaller c_3 values. Recent arguments and calculations have been made [13] that time correlation functions should decrease as a stretched exponential for the phase-turbulent regime of Eq. (1) in which case a correlation time can not be meaningfully defined. However, the change from exponential to stretched exponential behavior with decreasing c_3 has not yet been carefully studied. Shraiman et al conjectured that the critical scaling of τ at the L_1 line followed from the scaling of n_D with $\tau \propto 1/n_D$. Fig. 2(b) shows that this is approximately correct to the right of the L_1 line but breaks down closer to this line since the product τn_D is no longer constant: the defect density goes to zero faster than the correlation time increases.

In Fig. 3(a), we have calculated the phase spatial correlation length ξ_ϕ [16] as a function of c_3 . Shraiman et al argued that ξ_ϕ should be finite in the phase turbulent regime of Eq. (1) and estimated its value indirectly by calculating a phase diffusion coefficient $D = 1/\xi_\phi$ from phase-gradient correlations [2]. Exponential decay of spatial correlations is also expected for phase turbulence if the latter is described at long-wavelengths by the Kardar-Parisi-Zhang (KPZ) Langevin equation [13]. By going to quite large system sizes ($L = 10^6$) and to long integration times, we have verified directly that the phase spatial correlation function [16] decays exponentially well to the left of the L_1 -line as shown in Fig. 3(a). As the parameter c_3 decreases, the quantity ξ_ϕ varies *smoothly* through a local maximum near the L_1 line, and then increases steadily until we can no longer estimate its value accurately with our computer resources. The smooth variation of ξ_ϕ through the L_1 region is consistent with a crossover rather than with a sharp transition. The apparent divergence of ξ_ϕ upon approaching the Newell line, $\epsilon \rightarrow 0$, can be understood semiquantitatively as shown in Fig. 3(a) by a scaling argument [18] that predicts $\xi_\phi \propto \epsilon^{-5/2}$. The agreement is within about 10%.

The phase correlation length ξ_ϕ is the same as that of the field u itself [3], but there is a separate, generally shorter, correlation length scale ξ_ρ associated with fluctuations of the

field amplitude ρ (also with the phase gradient $\partial_x \phi$). Fig. 3(b) compares the reciprocals of the phase and amplitude correlation lengths with the Lyapunov dimension density δ , whose reciprocal defines the dimension correlation length ξ_δ discussed above [3]. Up to constant factor of 1.4, we find that the amplitude correlation length equals the dimension correlation length ξ_δ over a substantial range of parameter c_3 . (An independent and related result was also recently reported by other researchers [19].) *This remarkable result suggests that the big fractal dimension of some large homogeneous chaotic systems might be accurately estimated by simple correlation function calculations.* In work that we will discuss elsewhere [14], we have also calculated the variation of ξ_δ across a nonequilibrium Ising transition in a two-dimensional coupled map lattice [8]. Although the agreement is not quite so striking, ξ_δ still matches closely the correlation length associated with fluctuations of the magnitude of the site variables.

In Fig. 4, we make two final comparisons of how phase-turbulence, as described by the adiabatic approximation Eq. (3) and by solutions of the KS-equation Eq. (2), agrees with numerical solutions of Eq. (1). The dimension density δ of the KS-equation has been calculated to be $\delta = 0.230$ for the rescaled parameterless version of the KS-equation [20], $\partial_t \phi = -\partial_x^2 \phi - \partial_x^4 \phi - \phi \partial_x \phi$. Restoring the original space, time, and magnitude scalings gives the following c_1 and c_3 dependence of the dimension density for KS phase turbulence:

$$\delta = 0.230 \left(\frac{2(c_1 c_3 - 1)}{c_1^2 (1 + c_3^2)} \right)^{1/2}. \quad (5)$$

In Fig. 4(a), we compare Eq. (5) with our empirically determined values of δ for Eq. (1) from Fig. 3(b). The agreement is good up to about $c_3 = 0.5$ ($\epsilon = .75$) and then there is an increasing deviation of the actual solutions from Eq. (5). This deviation with increasing c_3 may arise because the adiabatic approximation Eq. (3) breaks down or because higher-order terms in the KS-equation are renormalizing the dimension density. Fig. 4(b) gives some further insight by comparing the mean-square fluctuation of ρ from Eq. (1) with the mean-square fluctuation of ρ as given by Eq. (3). We observe a previously unreported power-law scaling of these amplitude fluctuations with exponent $\alpha = 4$ from the Newell line to near

the L_1 line. Sufficiently close to the Newell line, an exponent of 4 is predicted by rescaling the solutions of Eq. (2). The adiabatic approximation is satisfied over a larger range in c_3 than the agreement between dimension densities.

In conclusion, we have used a powerful parallel computer to characterize more carefully the change from defect- to phase-turbulence near the L_1 line in the periodic one-dimensional Ginzburg-Landau equation in the limit of large system size. Instead of a sharp continuous transition with power-law scaling of order parameters [2], we found a smooth crossover with new and anomalous structure near the L_1 line, e.g., the variation of ξ_ϕ in Fig. 3(a) and the change in slopes of dimension density and amplitude fluctuations in Fig. 4. We confirmed recent predictions [2,13] that the phase correlation function decayed exponentially well to the left of the L_1 line, with the related correlation length being finite and large. We also found *two* length scales associated with the field u , a long scale associated with phase fluctuations and a short scale ξ_ρ associated with amplitude fluctuations. Surprisingly, the length ξ_ρ equals, up to a constant factor, the dimension correlation length ξ_δ associated with the dimension density. This suggests that spatial correlations of certain observables may suffice to estimate big fractal dimensions of some large homogeneous chaotic systems.

Numerous interesting questions remain for future study. Our calculations leave open the theoretical question of whether phase turbulence ($n_D = 0$) exists in the thermodynamic limit. More generally, it is still not known whether a chaos-to-chaos nonequilibrium transition can occur in an infinite one-dimensional system. Some of our results might be tested by experiment [10], e.g., the variation of phase correlation length (Fig. 3(a)) and the scaling of amplitude fluctuations near the Newell line (Fig. 4(b)). It would be interesting to extend our calculations to other parts of the c_1 - c_3 parameter plane, e.g., to understand the hysteretic bichaotic regime $c_1 < 1.8$, for which defect-turbulent and phase-turbulent states evidently coexist in parameter space [2,4]. If phase-turbulence is just a small n_D limit of defect-turbulence, it is more difficult to understand the coexistence of different states of identical symmetry. Finally, it would be interesting to repeat similar calculations in two- and three-space dimensions, for which the point-like space-time defects in one-space dimension are

replaced by long-lived topological defects such as vortices and lines [13].

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- [16] The correlation time τ of phase fluctuations was obtained by identifying graphically the region of asymptotic exponential decay $C(t) \propto \exp(-t/\tau)$ of the correlation function $C(t) = \langle e^{i[\phi(x, t+t') - \phi(x, t')]} \rangle$ where $e^{i\phi} = u/|u|$ is defined numerically in terms of a solution u of Eq. (1). The angle brackets denotes a space-time average over the region $(t', x) \in [0, T] \times [0, L]$. Similarly, the phase correlation length ξ_ϕ was obtained by identifying asymptotic exponentially decaying regions of the two-point function $C(x) = \langle e^{i[\phi(x+x', t) - \phi(x', t)]} \rangle$. Some correlation function plots are given in Refs [2] and [3].
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$e^{i\phi(x,t)}$ [16] decays exponentially with correlation length $\xi_\phi^{\text{KS}} = 9.6$. The desired scaling relation then follows by using Eqs (4) and (8) of Ref. [2] to establish a relation between the phase diffusion coefficients D of the KS- and CGL-equations. One finds that $\xi_\phi = \xi_\phi^{\text{KS}} f(c_1, c_3) \epsilon^{-5/2}$ with $f(c_1, c_3) = c_1(c_1 + c_3)^2 \sqrt{(1 + c_3^2)/2}$. This expression for ξ_ϕ is plotted in Fig. 3(a) for $c_1 = 3.5$.

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FIGURES

FIG. 1. **(a)** Log-log plot of the space-time defect density n_D versus the distance $c_3 - \bar{c}_3'$ to the fitted point \bar{c}_3' where the density goes to zero (onset of phase turbulence) for system size $L = 4096$, integration time $T = 10^5$, and an average over 64 randomly specified initial conditions. The arrow labeled “ L_1 ” indicates the position of the L_1 -line for parameter value $c_1 = 3.5$ [2]. The smallest n_D value corresponds to a count of 200 defects. The two solid lines were drawn to indicate the previous and present best estimates of the exponent α of a power-law scaling. The crosses are the data from Ref. [2]. In **(b)**, we find that Eq. 4 with exponent $\alpha = 1$ gives a better fit of the same data, with an onset of phase turbulence at $\bar{c}_3'' = 0.70$. The straight line is a plot of Eq. 4 over the range of its fit.

FIG. 2. **(a)** Plot of the reciprocal square root of the correlation time τ (as estimated from the asymptotic exponential decay of time correlation functions of the phase $e^{i\phi} = u/|u|$) versus the parameter c_3 . The arrow indicates where the L_1 -line occurs for $c_1 = 3.5$. We used a system size of $L = 4096$, an integration time of $T = 10^5$, and an average over 64 random initial conditions. The crosses denote the similar data from Ref. [2]. **(b)** For the same numerical parameters, a plot of the product of correlation time with defect density, τn_D , showing that the scaling $\tau \propto 1/n_D$ breaks down near the L_1 -line.

FIG. 3. **(a)** Plot of the phase correlation length ξ_ϕ for solutions of Eq. 1 for system sizes of up to $L = 10^6$, integration times of up to $T = 2 \times 10^5$, and averages 64 randomly chosen initial conditions. The crosses denote the similar data from Ref. [2]. The solid curve is the analytical expression obtained by scaling the finite correlation length of the parameterless KS-equation [18]. **(b)** Plot of the Lyapunov dimension density δ [3] and the reciprocals $1/\xi_\rho$ and $1/\xi_\phi$ of the amplitude and phase correlation lengths. The reciprocal length $1/\xi_\rho$ (open circles) has been scaled by a constant factor of 0.7 to emphasize the close agreement with δ . The positions of the Newell- and L_1 -lines for $c_1 = 3.5$ are denoted by the arrows labeled “N” and “ L_1 ” respectively.

FIG. 4. **(a)** Comparison of the dimension density δ for solutions of Eq. (1) with the rescaled dimension density of the KS-equation, Eq. (5), for $c_1 = 3.5$. **(b)** Comparison of the mean-square fluctuations of the amplitude ρ as calculated from the 1d CGL equation and as calculated from the adiabatic approximation, Eq. (3), with ϕ determined from Eq. (1. In both **(a)** and **(b)**, the arrows labeled “N” and “ L_1 ” denote the positions of the Newell- and L_1 -lines respectively for $c_1 = 3.5$.

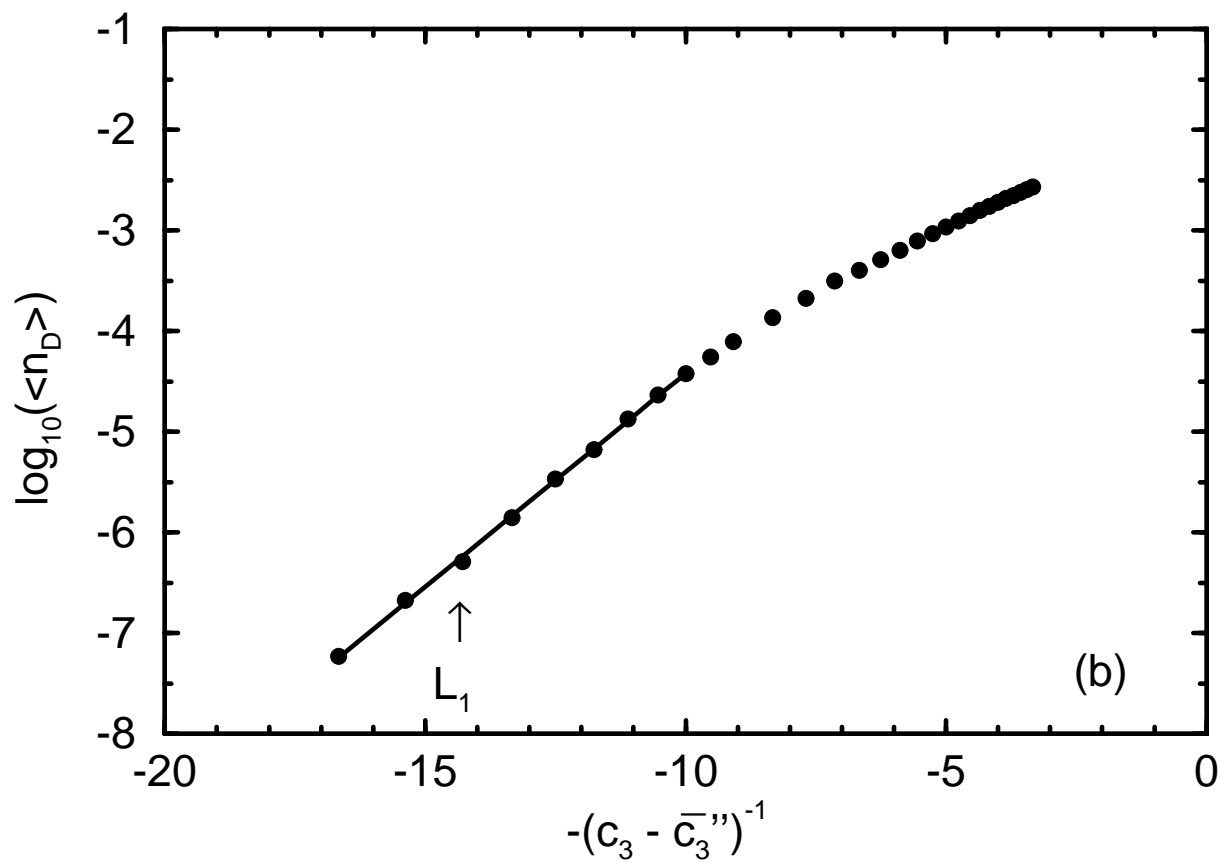
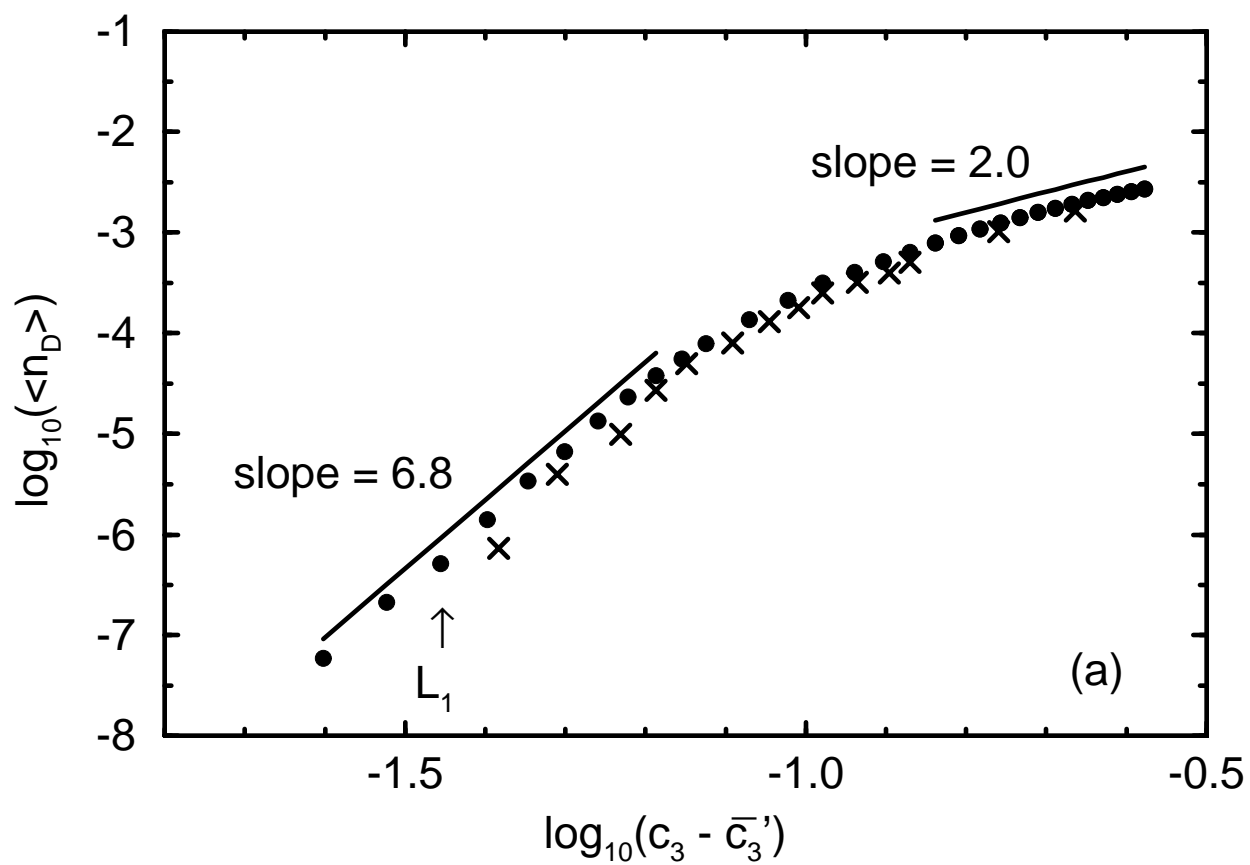


Fig. 1, Egolf and Greenside, "Characterization..."

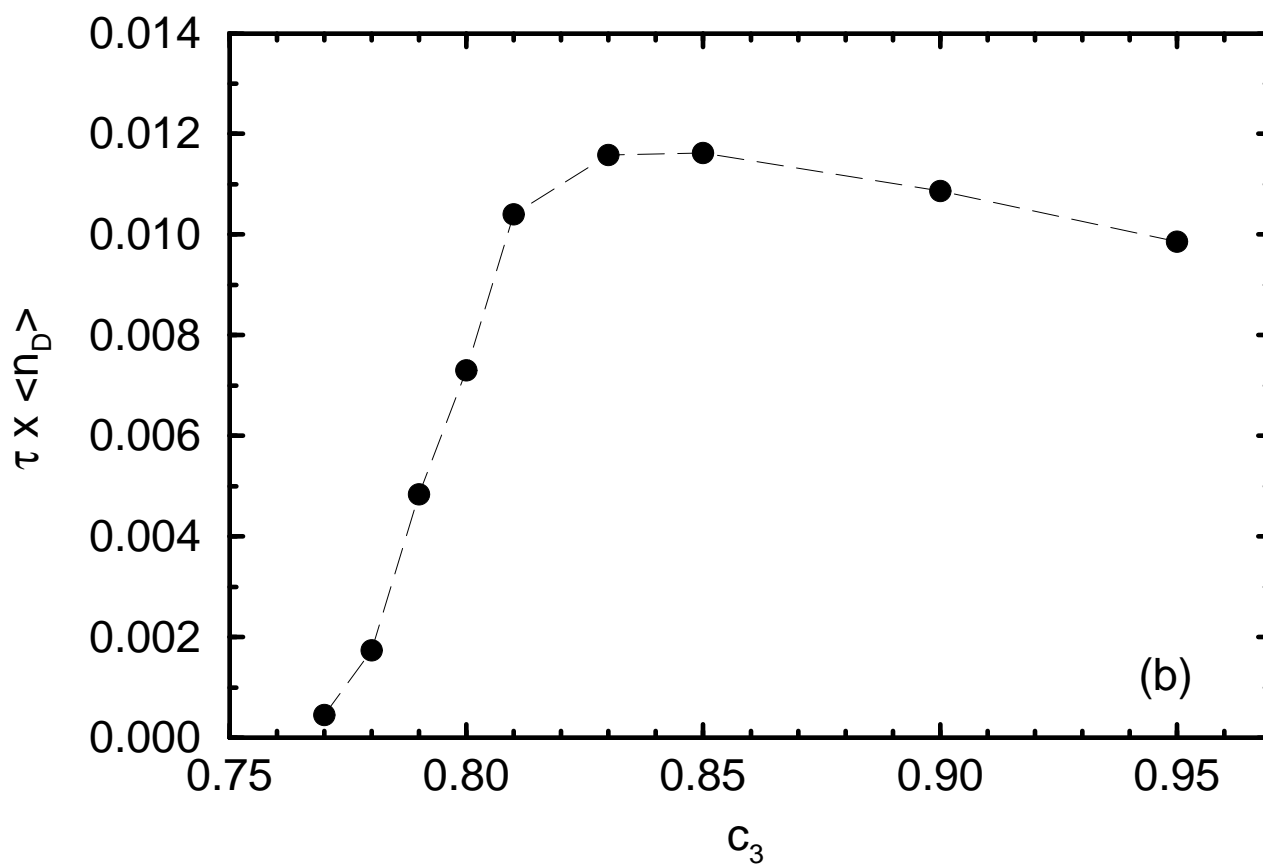
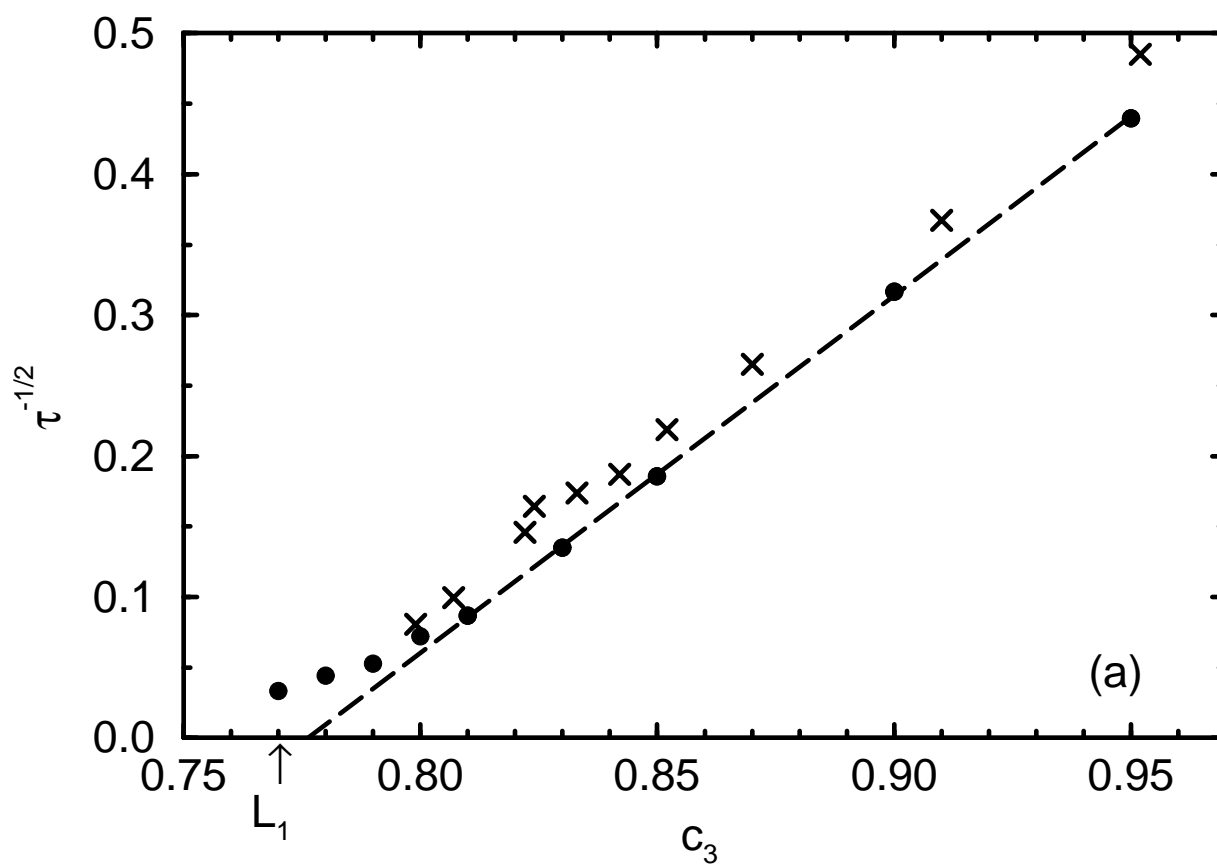


Fig. 2, Egolf and Greenside, "Characterization..."

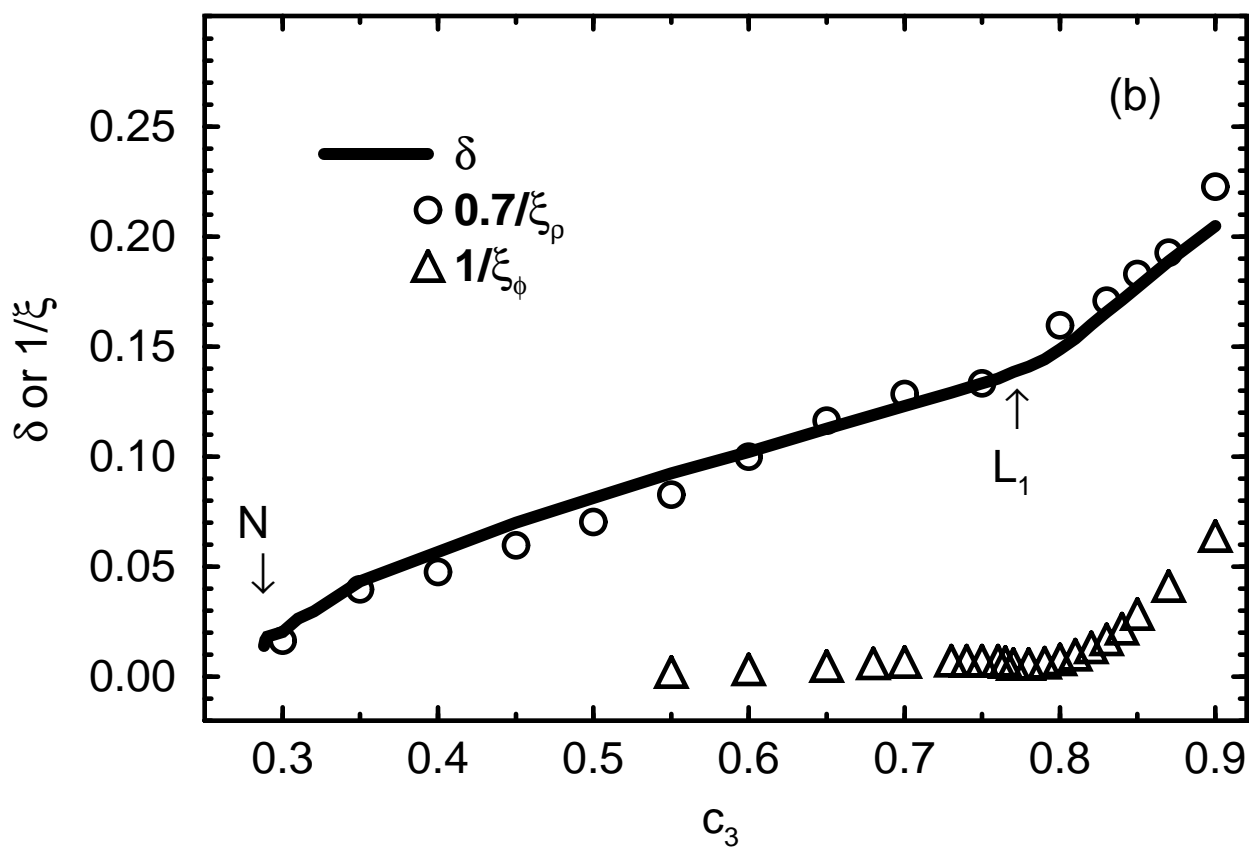
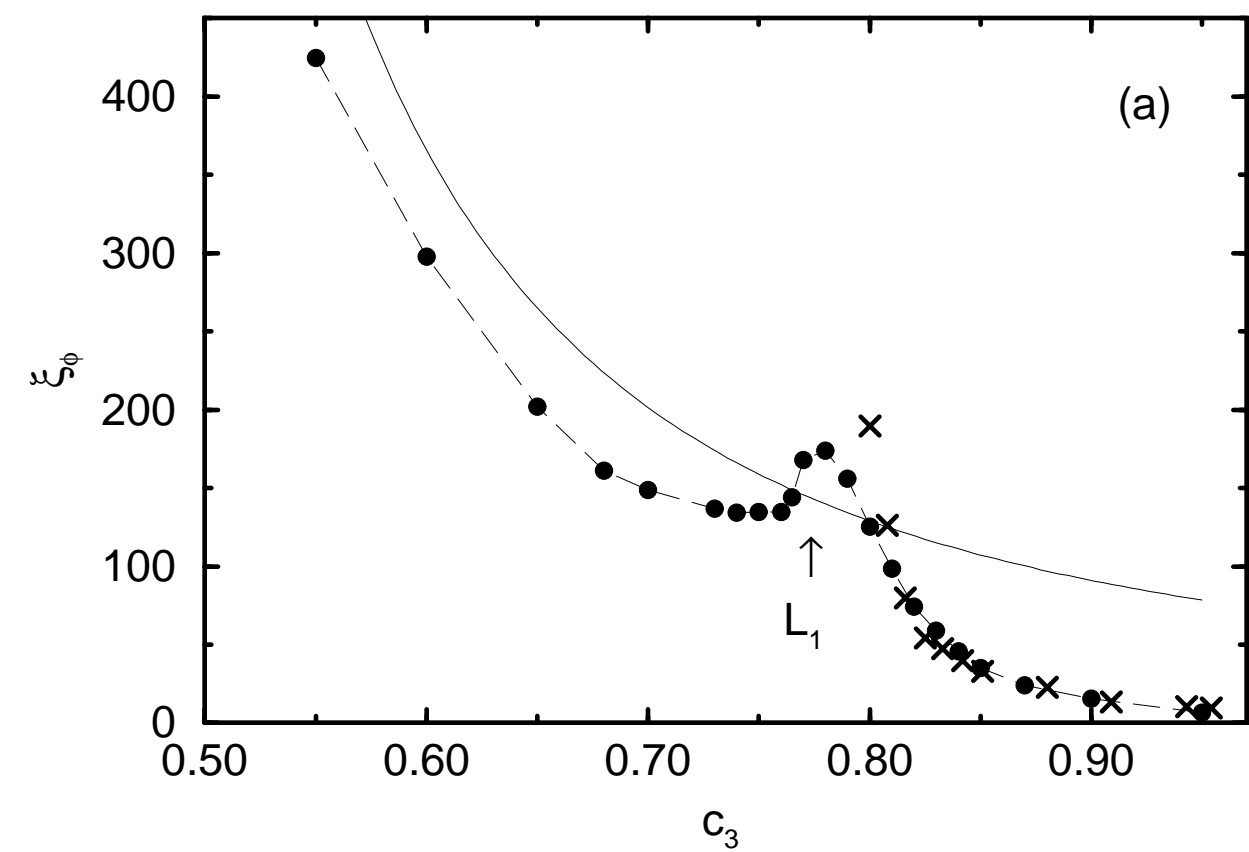


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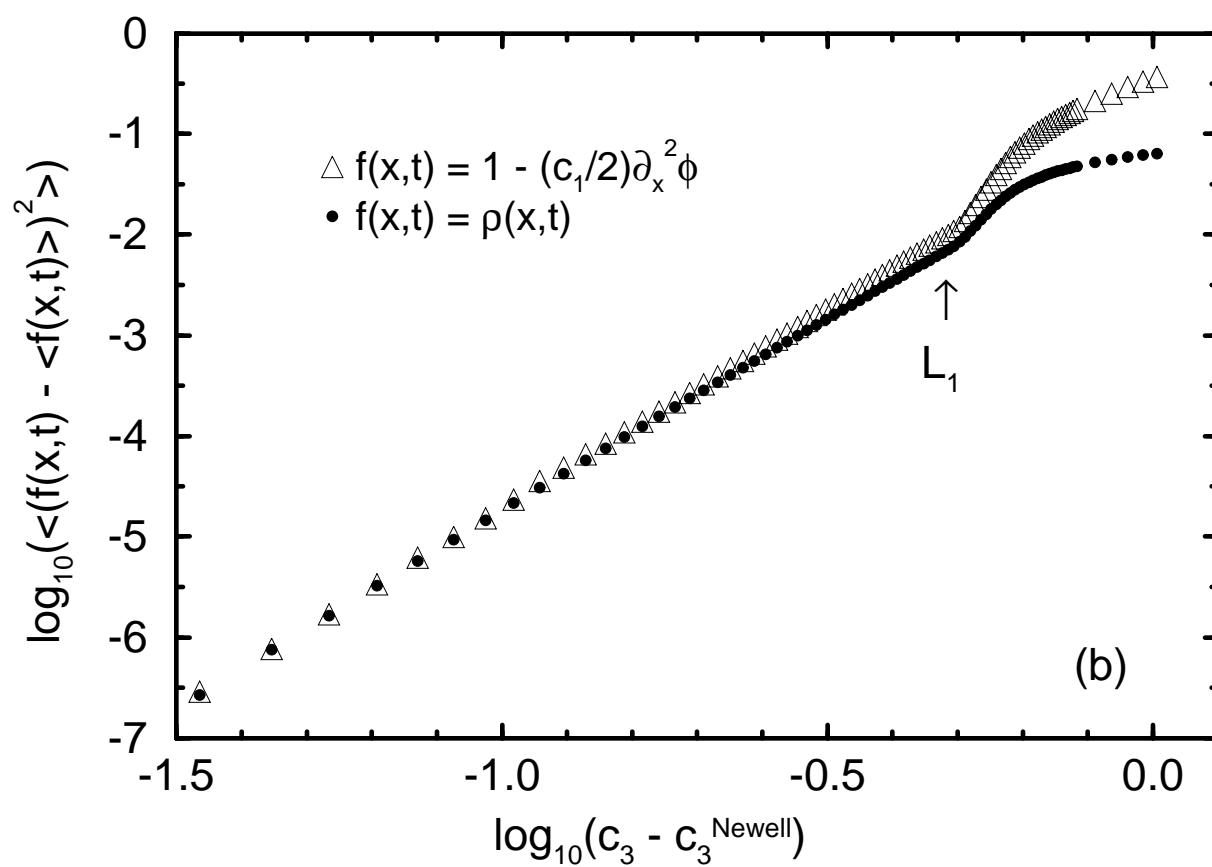
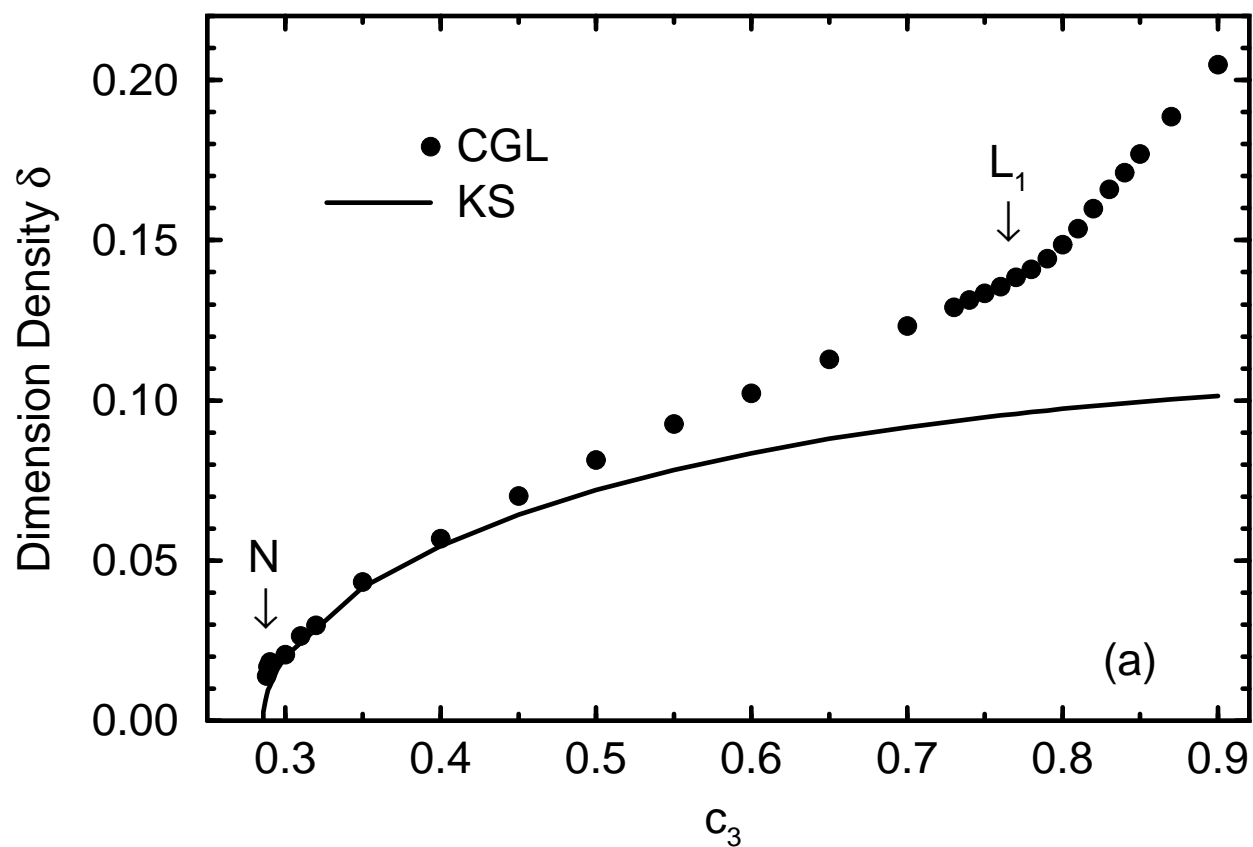


Fig. 4, Egolf and Greenside, "Characterization..."